EE221: Analog Electronics

Lecture Notes

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Contents

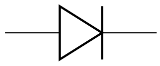
Ian Whiting (ian.whiting@usask.ca)

Electrical and Computer Engineering

1 Diodes

1.1 A Diodes Function

A diode is a one-way valve for electrical current. Current can flow in one direction, but is blocked flowing in the other direction.

(Anode)  (Cathode)

Where the arrow indicates the direction that current can flow.
Diodes operate in two states.

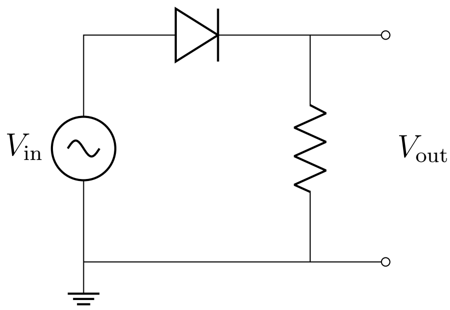
1. Forward Bias

   Anode voltage is sufficiently higher than cathode, and current is able to flow from the anode to the cathode.

2. Reverse Bias

   Cathode voltage is higher than anode, and only a tiny amount of current flows. Can be regarded as no current flowing for most circuits.

One of first applications for a diode was in rectifying AC signals. Hence why diodes are also called rectifiers.



1.2 Diode I-V Curve

To analyze diode circuits, we need an equation that relations the current through the diode, to the voltage applied.

$$I = I_S(e^{V/nV_T} - 1) \quad \text{Where} \quad V_T = \frac{kT}{e}.$$

I_S: Saturation current

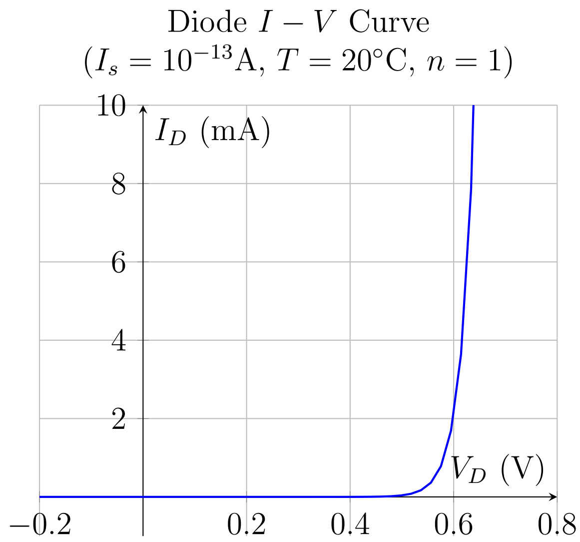
n: Ideality Factor ($1 < n < 2$)

T: Temperature

k: Boltzmann's constant ($8.617 \times 10^{-5} \frac{eV}{K}$)

e: Fundamental unit of charge

V_T: Thermal Voltage (25.25mV at 20° C)

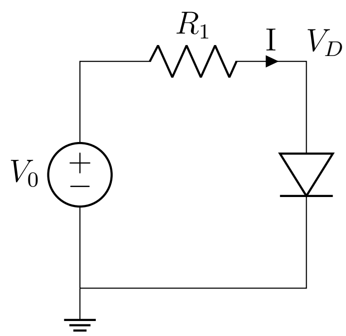


1.3 Diode Circuits

The Simplest diode circuit is a diode in series with a current limiting resistor. Where the resistor, R, limits the current in forward bias (Anode voltage > Cathode voltage).

1.3.1 Example —— Simple Diode Circuit

Solve the circuit for I and V_D.



KVL tells us that the sum of voltage drops around a closed loop must equal 0. As such, if we apply KVL.

$$V_0 - IR - V_D = 0.$$

To solve for current, we can express V_D in terms of I using the fundamental equation for a diode.

$$I = I_S(e^{V_D/nV_T} - 1).$$

Rearranging for V_D.

$$\frac{I}{I_S} = e^{V_D/nV_T} - 1$$
$$\frac{I}{I_S} + 1 = e^{V_D/nV_T}$$
$$\ln\left(\frac{I}{I_S} + 1\right) = \frac{V_D}{nV_T}$$
$$V_D = nV_T \ln\left(\frac{I}{I_S} + 1\right)$$

Thus,

$$V_0 - IR - nV_T \ln\left(\frac{I}{I_S} + 1\right) = 0.$$

We find that the equation above cannot be solved algebraically. As it turns out, the direct approach often does not work with non-linear components such as diodes.
What we can do is work out a numerical solution for the diode circuit.

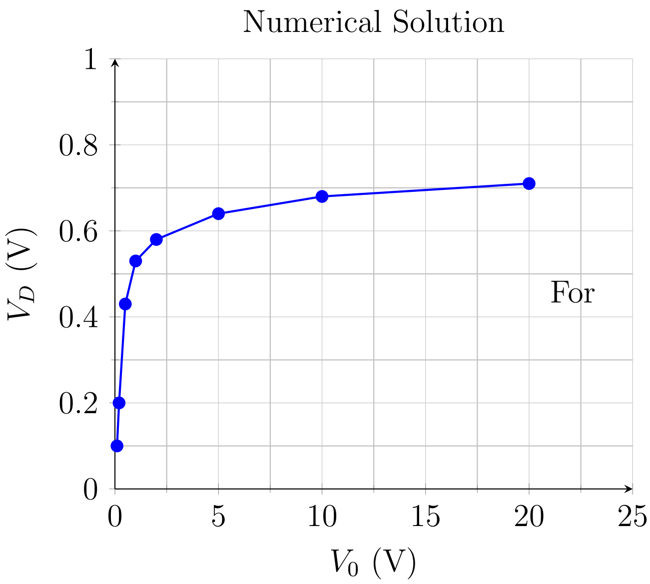
1.4 Numerical Solutions

To solve a diode circuit numerically, we need some information about the diode we are working with. Looking at the datasheet of a popular diode, the 1N914, we find that.

$$I_S = 25\text{nA} \qquad n = 2.$$

If we give a value to the resistor, say 500Ω, and allow the voltage source to sweep from 0.1V — 20V we find the following.

V_0 (V)	I (mA)	V_D (V)
0.1	0.00016	0.10
0.2	0.0016	0.20
0.5	0.138	0.43
1	0.95	0.53
2	2.84	0.58
5	8.72	0.64
10	18.7	0.68
20	38.6	0.71



For $V_0 < 0.5$V, we can see that the current is limited to low values by the diode and there is little voltage drop across the resistor.
For $V_0 > 2$V, the voltage drop across the diode does not change much going from 0.6V — 0.7V. Most of V_0 is dropped across the resistor.
Numerical solutions are quick and accurate, but the solution is specific to the circuit you are working on, and thus needs to be repeated for each and every circuit.

1.5 Graphical Solution

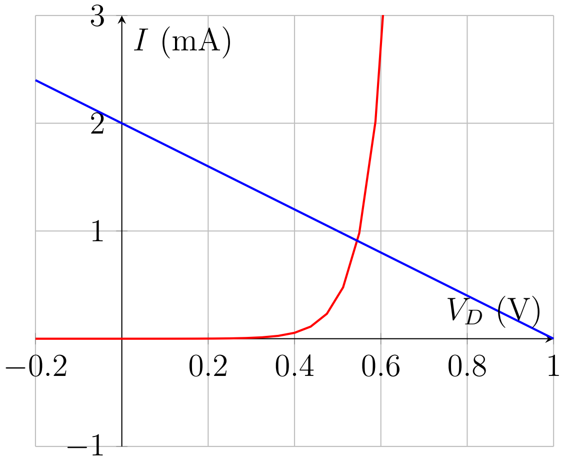
Graphical solutions are a method of solving a circuit visually, by looking at the intersection points of two traces on a graph. However, this method is inaccurate.
The flip side is that it provides insight into how the circuit works.
To solve a circuit graphically:

1. Graph I vs V_D for the diode.
2. Graph the I—V relation for the resistor.

To graph the I—V relationship, we need to equate V_R to V_D.

From KVL: $V_R = V_0 - V_D$
From Ohm's Law: $I = V_R/R = (V_0 - V_D)/R$

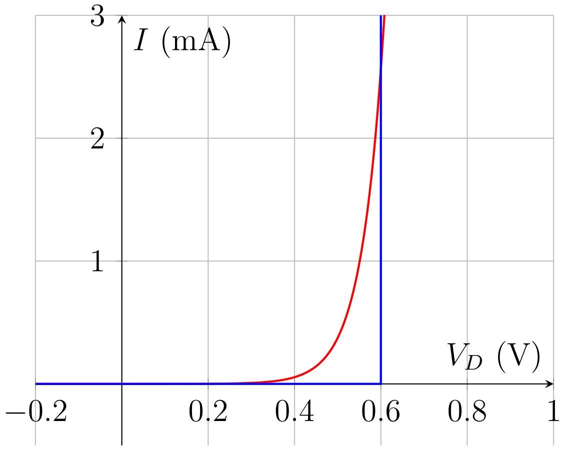
Referencing the simple diode circuit example, we come up with the following plot.



The solution can be read from where the two traces intersect.
The blue line is called the <u>load line</u>. This is because the resistor is asking as the output load for the diode.
The y-intercept is V_0/R and the x-intercept is simply V_0.
The load line changes as either V_0 or R changes.
Increasing V_0 increases both intercepts proportionately. As V_0 increases the x-intercept increase and thus the intercept of the two traces moves upwards.
Increasing R reduces the y-intercept but not the x-intercept, thus deducing the slope.

1.6 Diode Approximation

We can approximate the diodes I-V curve by a simpler function.
Where the current increases rapidly, we can say that the diode looks like a voltage source with voltage V_F.
For small currents, the diode looks like an open connection.



In the above graph we approximate the true I-V curve in red with the curve in blue. There are two assumptions we make.

$$\text{For forward bias:} \quad I > 0 \quad \text{and} \quad V_D = V_F$$
$$\text{For reverse bias:} \quad I = 0 \quad \text{and} \quad V_D < V_F$$

The value for V_F depends on the diode and the typical current in forward bias.
For diodes in low current circuits, V_F is usually 0.6—0.7V.
For rectifiers in high current power circuits, V_F can be 0.8V or higher.
The steps to use diode approximation are as follows:

1. If the diode is reverse-biased, replace the diode with an open circuit.
2. If the diode is forward-biased, replace the diode with a voltage source.

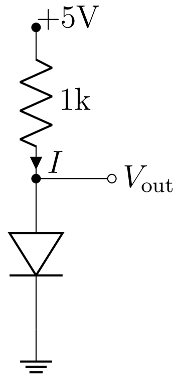
Now we are able to solve the circuit using linear circuit analysis. However, we need to determine if the diode is forward or reverse biased.
To determine this we need to solve the circuit in two ways.

1. Assume reverse-biased, and replace the diode with an open.
2. Solve the circuit.
3. Check if assumptions are true ($I = 0$, and $V_D < V_F$)
4. Assume forward-biased, and replace diode with a voltage source.
5. Solve the circuit.
6. Check if assumptions are true ($I > 0$, and $V_D = V_F$)

If there is more than one diode, you must check all combinations. Meaning for a circuit with 2 diodes you need to check D_1: forward & reverse, D_2: forward & reverse.
The correct solution is the solution that is consistent for both.

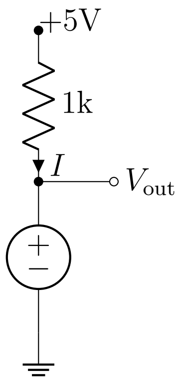
1.6.1 Example —— Simple Diode Direction 1

For the circuit below, find the voltage at V_{out}. Use $V_F = 0.7$V.



First we will assume that the diode is forward biased. Meaning, we assume the anode is at a higher voltage than the cathode.
If this assumption is correct, we will find the following two conditions to be true: $I > 0$ and $V_D = V_F$. Where V_D is the voltage drop across the diode.

1. Assume Forward Biased If the diode is forward biased, we can replace it with voltage source, V_F.



Solving the circuit using KVL, we see the following:

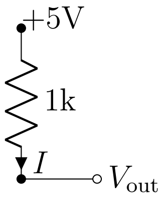
$$V_S - V_R - V_F = 0$$
$$5V - V_R - 0.7V = 0$$
$$V_R = 4.3V$$

If the voltage drop across the resistor is $4.3V$, then the voltage at V_{out} must be $5V - 4.3V = 0.7V$.
Similarly, we can find the current through Ohm's law.

$$I_R = \frac{4.3V}{1K\Omega} = 4.3\text{mA}.$$

Checking the conditions we see that $I > 0$ is true, and $V_D = V_F$ is also true.
Even though in this question it is obvious the diode is forward biased, we can prove it by assuming the opposite and checking conditions.

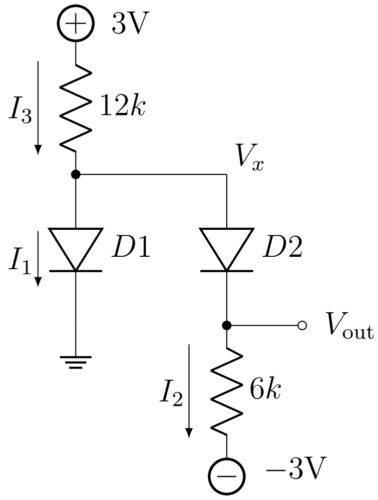
2. Assume Reverse Biased If the diode is reverse biased, we can replace it with an open circuit. Thus, our circuit becomes:



From this, it is clear to see that no current flows, and V_{out} sees the entire $5V$ that is being supplied.
Comparing these against the conditions required for a diode to be reverse biased, $I = 0$ is true but $V_D < V_F$ is false as $5V > 0.7V$.

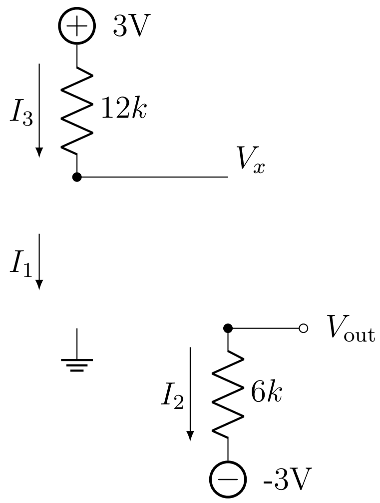
1.6.2 Example —— Two Diodes

For the circuit below, find V_{out}. Assume $V_F = 0.7V$.



In this example there are two diodes. For each diode we need to assume a forward bias and a reverse bias, checking conditions for each. Thus, we have the four cases:

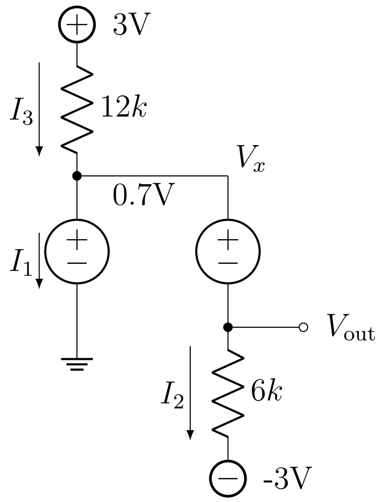
1. Case 1: D_1 reverse, D_2 reverse.



Assuming a reverse bias for each diode, we replace the diodes with open circuits.
If the diodes are replaced with open circuits, we need to find the voltage drop across each diode *as if they were still there.*
So, for D_1 the voltage drop is 3V as V_x is at a potential of 3V due to no current flowing from the +3V source.
Similarly, for D_2 we check the voltage across the diode as being $3V - (-3V) = 6V$.
Checking these against our conditions,

$$V_{D_1} = 3V < V_F \Rightarrow \text{ Inconsistent}$$
$$V_{D_2} = 6V < V_F \Rightarrow \text{ Inconsistent}$$

2. Case 2: D_1 forward, D_2 forward.



Assuming a forward bias for each diode, we replace the diodes with a voltage source, V_F. Solving this circuit we find that V_x is at a voltage of $0.7V$. Similarly, V_{out} is at a voltage of $0V$. This is because the diodes, which were replaced by voltage sources, hold those nodes at the voltages dictated by the forward voltage of the diode.

Given that V_x is at $0.7V$, the current through I_3 is given by.

$$I_3 = (2.3V)/(12k\Omega) = 0.192\text{mA}.$$

Similarly, for I_2 we find

$$I_2 = (3V)/(6k\Omega) = 0.5\text{mA}.$$

By KCL, we find I_1.

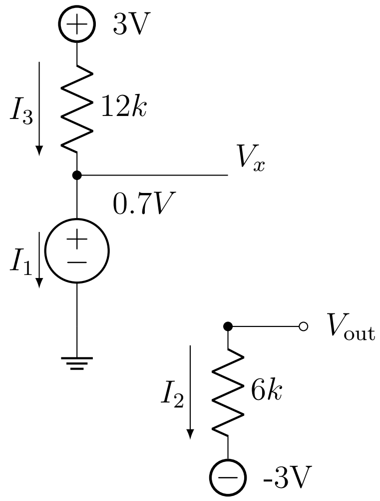
$$I_3 - I_1 - I_2 = 0$$
$$I_1 = I_3 - I_2$$
$$I_1 = -3.08\text{mA}$$

Now that we know the current through all of the diodes we can check the conditions for diodes in forward bias, $I > 0$.

$$I_{D_1} = -3.08\text{mA} \Rightarrow \text{ Inconsistent}$$
$$I_{D_2} = 0.5\text{mA} \Rightarrow \text{ Consistent}$$

Both conditions are not met, therefore this is not our solution.

3. Case 3: D_1 forward, D_2 reverse.



If D_1 is forward biased, we replace it with a voltage source of $V_F = 0.7V$, D_2 we replace with an open circuit.

The conditions we need to check is that D_1 has a positive current flowing, and the drop across it is equal to V_F.

For D_2 we need to check that there is no current flowing, and that the drop across it is less than V_F.

Replacing D_1 tells us that V_x is at a potential of $0.7V$. From this, we can find the current through the resistor.

$$I_3 = I_1 = (2.3V)/(12k\Omega) = 0.192\text{mA}.$$

The voltage across D_2 is then,

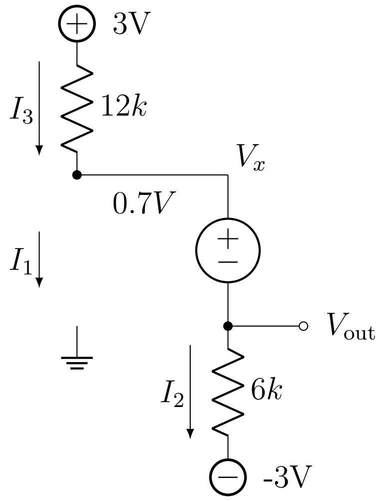
$$V_x - (-3V) = 3.7V.$$

Checking our conditions we find,

$$I_1 = 0.192\text{mA} \Rightarrow \text{ Consistent}$$
$$V_{D_2} = 3.7V > V_F \Rightarrow \text{ Inconsistent}$$

Therefore, this is also not our solution.

4. Case 4: D_1 reverse, D_2 forward.



Rapid fire through this one as it is pretty similar to the last.
One notable difference is that V_x is not simply at $0.7V$.

$$3V - V_{R_1} - V_F - V_{R_2} - (-3V) = 0$$
$$I(R_1 + R_2) = 6V - V_F$$
$$I(18k\Omega) = 5.3V$$
$$I = 0.294\text{mA}$$

Meaning that V_x is at a potential of

$$3V - (0.294\text{mA})(12k\Omega) = -0.528V.$$

Checking against our conditions we find,

$$V_{D_1} = V_x = -0.528V < V_F \Rightarrow \text{ Consistent}$$
$$I_{D_1} = I_2 = 0.294\text{mA} \Rightarrow \text{ Consistent}$$

Thus, this is our solution.